Mr. Wright's Math Extravaganza

Physical Sciences

(Chemistry, Physics, Physical Science)

Force

Units 02 Newton's Laws, 03 Uniform Circular Motion, 04 Momentum

Average Level for All Three Units

Level 2.0: 70% on test, Level 3.0: 80% on test, Level 4.0: level 3.0 and success on bumper lab

Score	I Can Statements
4.0	☐ I can design a device that minimizes force on an object during a collision and justify its design.
3.5	In addition to score 3.0 performance, partial success at score 4.0 content.
3.0	**02 Newton's Laws** ☐ I can use Newton's second law of motion to describe the mathematical relationships between net force, acceleration, and mass. **03 Uniform Circular Motion** ☐ I can explain how unbalanced forces applied to a system can cause a change in its rotational motion. **04 Momentum** ☐ I can explain how to minimize force on an object during a collision. ☐ I can explain why the total momentum of a system of objects is conserved when there is no net force on the system.
2.5	No major errors or omissions regarding score 2.0 content, and partial success at score 3.0 content.
2.0	**02 Newton's Laws** ☐ I can explain why force and acceleration are often represented with vectors. ☐ I can defend Newton's first law of motion by explaining what a balanced net force of zero means when related to objects in motion and at rest. ☐ I can explain the difference between mass and weight and their common measurement units. ☐ I can recall the equation for Newton's second law of motion. ☐ I can explain Newton's third law of motion. **03 Uniform Circular Motion** ☐ I can calculate torque on a rotating object. ☐ I can calculate how changes in the moment of inertia cause changes to its rotational velocity. **04 Momentum** ☐ I can use Newton's second law of motion and the equation for acceleration to find the relationship between impulse and momentum change. ☐ I can explain the inverse relationship between force and time using the equations for impulse and momentum change. ☐ I can explain why objects colliding for the same amount of time experience equal impulse in opposite directions, and therefore equal and opposite changes in momentum. ☐ I can recall the law of conservation of momentum. ☐ I can compare the initial and final momenta of objects in a collision.

	☐ I can determine the degree to which a collision is elastic or inelastic by determining whether kinetic energy is conserved. ☐ I can assess the force, momentum, impulse, and velocity associated with real-world examples of the rebound effect and elastic and inelastic collisions.
1.5	Partial success at score 2.0 content, and major errors or omissions regarding score 3.0 content.
1.0	With help, partial success at score 2.0 content and score 3.0 content.
0.5	With help, partial success at score 2.0 content but not at score 3.0 content.
0.0	Even with help, no success.

Physics 04-01 Impulse and Momentum **Name: ________________________**

Impulse

$$J = F\Delta t$$

Unit: Ns

Is a ___________________

Both ___________________ and ___________________ play role in how responds to ___________________

Linear Momentum

$$p = mv$$

Unit: kg m/s

Is a ___________________

Is important when talking about ___________________

Impulse-Momentum Theorem

$$\boldsymbol{F\Delta t = mv_f - mv_0}$$

___________________ = change in ___________________

Find change in momentum

- Use ______________________________________ and _______________of contact to find average ________________ of contact

A baseball (m = 0.14 kg) with initial velocity of −40 m/s (90 mph) is hit. It leaves the bat with a velocity of 60 m/s after 0.001 s. What is the impulse and average net force applied to the ball by the bat?

A raindrop (m = 0.001 kg) hits a roof of a car at −15 m/s. After it hits, it spatters so the effective final velocity is 0. The time of impact is 0.01 s. What is the average force?

What if it is ice so that it bounces off at 10 m/s?

Physics 04-01 Impulse and Momentum	*Name: ________________________*
Practice Work	

1. Two identical automobiles have the same speed, one traveling east and one traveling west. Do these cars have the same momentum? Explain.
2. Two objects have the same momentum. Do the velocities of these objects necessarily have (a) the same directions and (b) the same magnitudes? Give your reasoning in each case.
3. You have a choice. You may get hit head-on either by an adult moving slowly on a bicycle or by a child that is moving twice as fast on a bicycle. The mass of the child and bicycle is one-half that of the adult and bicycle. Considering only the issues of mass and velocity, which collision do you prefer? Or doesn't it matter? Account for your answer.
4. When you're driving a golf ball, a good "follow-through" helps to increase the distance of the drive. A good follow-through means that the club head is kept in contact with the ball as long as possible. Using the impulse-momentum theorem, explain why this technique allows you to hit the ball farther.
5. Explain in terms of impulse how padding reduces forces in a collision. State this in terms of a real example, such as the advantages of a carpeted vs. tile floor for a day care center.
6. (a) Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of 7.50 m/s. (b) Compare the elephant's momentum with the momentum of a 0.0400-kg tranquilizer dart fired at a speed of 600 m/s. (c) What is the momentum of the 90.0-kg hunter running at 7.40 m/s after missing the elephant? (OpenStax 8.1) $\mathbf{1.50 \times 10^4}$ **kg m/s, 24.0 kg m/s,** $\mathbf{6.66 \times 10^2}$ **kg m/s**
7. (a) What is the mass of a large ship that has a momentum of 1.60×10^9 kg · m/s, when the ship is moving at a speed of 48.0 km/h? (b) Compare the ship's momentum to the momentum of a 1100-kg artillery shell fired at a speed of 1200 m/s. (OpenStax 8.2) $\mathbf{1.20 \times 10^8}$ **kg,** $\mathbf{1.32 \times 10^6}$ **kg m/s**
8. A bullet is accelerated down the barrel of a gun by hot gases produced in the combustion of gun powder. What is the average force exerted on a 0.0300-kg bullet to accelerate it to a speed of 600 m/s in a time of 2.00 ms (milliseconds)? (OpenStax 8.7) $\mathbf{9.00 \times 10^3}$ **N**
9. A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seat belt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg. (OpenStax 8.8) **2690 N**
10. Suppose a child drives a bumper car head on into the side rail, which exerts a force of 4000 N on the car for 0.200 s. (a) What impulse is imparted by this force? (b) Find the final velocity of the bumper car if its initial velocity was 2.80 m/s and the car plus driver have a mass of 200 kg. You may neglect friction between the car and floor. (OpenStax 8.11) **–800 kg m/s, –1.20 m/s**
11. One hazard of space travel is debris left by previous missions. There are several thousand objects orbiting Earth that are large enough to be detected by radar, but there are far greater numbers of very small objects, such as flakes of paint. Calculate the force exerted by a 0.100-mg chip of paint that strikes a spacecraft window at a relative speed of 4.00×10^3 m/s, given the collision lasts 6.00×10^{-8} s. (OpenStax 8.12) $\mathbf{6.67 \times 10^3}$ **N**
12. A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment. (a) Calculate the average force on the person if he is stopped by a padded dashboard that compresses an average of 1.00 cm. (b) Calculate the average force on the person if he is stopped by an air bag that compresses an average of 15.0 cm. (OpenStax 8.13) $\mathbf{-1.50 \times 10^6}$ **N,** $\mathbf{-1.00 \times 10^5}$ **N**
13. A volleyball is spiked so that its incoming velocity of +4.0 m/s is changed to an outgoing velocity of −21 m/s. The mass of the volleyball is 0.35 kg. What impulse does the player apply to the ball? (Cutnell 7.1) **–8.8 kg m/s**
14. A baseball (m = 149 g) approaches a bat horizontally at a speed of 40.2 m/s (90 mph) and is hit straight back at a speed of 45.6 m/s (102 mph). If the ball is in contact with the bat for a time of 1.10 ms, what is the average force exerted on the ball by the bat? Neglect the weight of the ball, since it is so much less than the force of the bat. Choose the direction of the incoming ball as the positive direction. (Cutnell 7.4) **–11600 N**

Physics 04-02 Conservation of Momentum | *Name: ________________________*

System

All the ________________ involved in the ________________
Usually only ________________ objects

________________ Forces – Forces that the objects exert on each other
________________ Forces – Forces exerted by things outside of the system

Law of Conservation of Momentum

In an ________________ system the total momentum remains ________________

$$\boldsymbol{p_0} = \boldsymbol{p_f}$$

Reasoning Strategy

1. Decide on the ________________
2. Identify ________________ and ________________ ________________
3. Is the system ________________? If no, then ________________ use conservation of momentum
4. Set the total ________________ momentum of the isolated system ________________ to the total ______________ momentum

Two billiard balls are colliding on a table. In order to apply the law of conservation of momentum, what should the system be? One ball or both billiard balls?

A hockey puck of mass 0.17 kg and velocity 5 m/s is caught by a 0.5 kg mitten laying on the ice. What is the combined velocity after the puck is in the mitten? (ignore friction)

A 5 kg baseball pitching machine is placed on some frictionless ice. It shoots a 0.15 kg baseball horizontally at 35 m/s. How fast is the pitching machine moving after it shoots the ball?

Practice Work

1. In movies, Superman hovers in midair, grabs a villain by the neck, and throws him forward. Superman, however, remains stationary. Using the conservation of linear momentum, explain what is wrong with this scene.
2. A satellite explodes in outer space, far from any other body, sending thousands of pieces in all directions. How does the linear momentum of the satellite before the explosion compare with the total linear momentum of all the pieces after the explosion? Explain.
3. You are a passenger on a jetliner that is flying at a constant velocity. You get up from your seat and walk toward the front of the plane. Because of this action, your forward momentum increases. What, if anything, happens to the forward momentum of the plane? Give your reasoning.
4. An ice boat is coasting along a frozen lake. Friction between the ice and the boat is negligible, and so is air resistance. Nothing is propelling the boat. From a bridge someone jumps straight down and lands in the boat, which continues to coast straight ahead. (a) Does the horizontal momentum of the boat change? (b) Does the speed of the boat increase, decrease, or remain the same? Explain.
5. A 55-kg swimmer is standing on a stationary 210-kg floating raft. The swimmer then runs off the raft horizontally with a velocity of +4.6 m/s relative to the shore. Find the recoil velocity that the raft would have if there were no friction and resistance due to the water. (Cutnell 7.16) **−1.2 m/s**
6. Two friends, Al and Jo, have a combined mass of 168 kg. At an ice skating rink they stand close together on skates, at rest and facing each other, with a compressed spring between them. The spring is kept from pushing them apart because they are holding each other. When they release their arms, Al moves off in one direction at a speed of 0.90 m/s, while Jo moves off in the opposite direction at a speed of 1.2 m/s. Assuming that friction is negligible, find Al's mass. (Cutnell 7.18) **96 kg**
7. In a science fiction novel two enemies, Bonzo and Ender, are fighting in outer space. From stationary positions they push against each other. Bonzo flies off with a velocity of +1.5 m/s, while Ender recoils with a velocity of −2.5 m/s. (a) Without doing any calculations, decide which person has the greater mass. Give your reasoning. (b) Determine the ratio of the masses (m_{Bonzo}/m_{Ender}) of these two people. (Cutnell 7.17) **1.7**
8. Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 150,000 kg and a velocity of 0.300 m/s, and the second having a mass of 110,000 kg and a velocity of −0.120 m/s. (The minus indicates direction of motion.) What is their final velocity? (OpenStax 8.23) **0.122 m/s**
9. Suppose a clay model of a koala bear has a mass of 0.200 kg and slides on ice at a speed of 0.750 m/s. It runs into another clay model, which is initially motionless and has a mass of 0.350 kg. Both being soft clay, they naturally stick together. What is their final velocity? (OpenStax 8.24) **0.273 m/s**
10. Consider the following question: *A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seatbelt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.* Would the answer to this question be different if the car with the 70-kg passenger had collided with a car that has a mass equal to and is traveling in the opposite direction and at the same speed? Explain your answer. (OpenStax 8.25)
11. What is the velocity of a 900-kg car initially moving at 30.0 m/s, just after it hits a 150-kg deer initially running at 12.0 m/s in the same direction? Assume the deer remains on the car. (OpenStax 8.26) **27.4 m/s**
12. A 1.80-kg falcon catches a 0.650-kg dove from behind in midair. What is their velocity after impact if the falcon's velocity is initially 28.0 m/s and the dove's velocity is 7.00 m/s in the same direction? (OpenStax 8.27) **22.4 m/s**

Physics 04-03 Elastic and Inelastic Collisions — Name: ______________________

Kinetic Energy

- Energy of **motion**
- $KE = \frac{1}{2}mv^2$

Subatomic – **kinetic** energy often **conserved**

Macroscopic – **kinetic** energy usually not **conserved**

- Converted into **heat**
- Converted into **distortion** or **damage**

Elastic – **kinetic** energy **conserved**

Inelastic – **kinetic** energy **Not conserved**

Completely inelastic – the objects **stick** together

You are playing marbles. Your 0.10 kg shooter traveling at 1 m/s hits a stationary 0.05 kg cat's eye marble. If it is an elastic collision what are the velocities after the collision?

Momentum

$$m_s v_{0s} + m_c v_{0c} = m_s v_{fs} + m_c v_{fc}$$

$$(0.1\ kg)\left(1\frac{m}{s}\right) + (0.05\ kg)(0) = (0.1\ kg)v_{fs} + (0.05\ kg)v_{fc}$$

$$0.1\ kg\frac{m}{s} = (0.1\ kg)v_{fs} + (0.05\ kg)v_{fc}$$

$$v_{fs} = 1\ m/s - 0.5\ v_{fc}$$

Kinetic Energy

$$\frac{1}{2}m_s v_{0s}^2 + \frac{1}{2}m_c v_{0c}^2 = \frac{1}{2}m_s v_{fs}^2 + \frac{1}{2}m_c v_{fc}^2$$

$$\frac{1}{2}(0.1\ kg)\left(1\frac{m}{s}\right)^2 + 0 = \frac{1}{2}(0.1\ kg)v_{fs}^2 + \frac{1}{2}(0.05\ kg)v_{fc}^2$$

$$0.05\ J = (0.05\ kg)v_{fs}^2 + (0.025\ kg)v_{fc}^2$$

$$v_{fs}^2 + 0.5\ v_{fc}^2 = 1\left(\frac{m}{s}\right)^2$$

Substitution

$$\left(1\frac{m}{s} - 0.5v_{fc}\right)^2 + 0.5\ v_{fc}^2 = 1\left(\frac{m}{s}\right)^2$$

$$1\left(\frac{m}{s}\right)^2 - \left(1\frac{m}{s}\right)v_{fc} + 0.25\ v_{fc}^2 + 0.5\ v_{fc}^2 = 1\left(\frac{m}{s}\right)^2$$

$$-\left(1\frac{m}{s}\right)v_{fc} + 0.75\ v_{fc}^2 = 0$$

$$v_{fc}\left(-1\frac{m}{s} + 0.75\ v_{fc}\right) = 0$$

$$v_{fc} = 0\ or\ 1.33\ m/s$$

$$v_{fs} = 1\frac{m}{s} - 0.5\ v_{fc}$$

$$V_{fs} = 1\ m/s - 0.5(1.33\ m/s) = 0.333\ m/s$$

Police will sometime reconstruct car accidents. In one accident, the cars stuck together and slid 12 m before they stopped. They measure the coefficient of friction as 0.70. The blue car's mass is 1100 kg and was sitting still at a stop sign when it was hit by the red car whose mass is 990 kg. How fast was the red car going when it hit the blue car?

Work backwards

Find acceleration after collision:

$$v^2 = v_0^2 + 2a(d - d_0)$$

$$0^2 = v_0^2 + 2a(12\ m - 0m)$$

$$-\frac{v_0^2}{24\ m} = a$$

Forces (cars stuck together m=2090 kg):

$$\mu F_N = ma$$

$$-(0.70)(mg) = ma$$

$$-(0.70)\left(9.8\frac{m}{s^2}\right) = -\frac{v_0^2}{24\ m}$$

$$164.64\frac{m^2}{s^2} = v_0^2$$

$$v_0 = 12.83\frac{m}{s}$$

Use momentum (v_0 is now final v):

$$m_1 v_{01} + m_2 v_{02} = m_1 v_1 + m_2 v_2$$

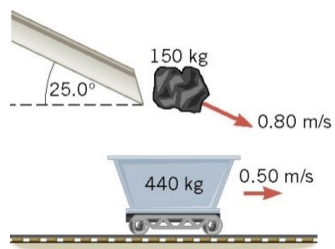
$$(1100\ kg)\left(0\frac{m}{s}\right) + (990\ kg)v_{02} = (1100\ kg)\left(12.83\frac{m}{s}\right) + (990\ kg)\left(12.83\frac{m}{s}\right)$$

$$(990\ kg)v_{02} = 26817.2\ kg\frac{m}{s}$$

$$v_{02} = 27.1\frac{m}{s}$$

Physics 04-03 Elastic and Inelastic Collisions	*Name: ________________________*
Practice Work	

1. In an elastic collision, is the kinetic energy of *each* object the same before and after the collision? Explain.
2. What is an elastic collision?
3. What is an inelastic collision? What is a perfectly inelastic collision?
4. Mixed-pair ice skaters performing in a show are standing motionless at arms length just before starting a routine. They reach out, clasp hands, and pull themselves together by only using their arms. Assuming there is no friction between the blades of their skates and the ice, what is their velocity after their bodies meet?
5. In a football game, a receiver is standing still, having just caught a pass. Before he can move, a tackler, running at a velocity of +4.5 m/s, grabs him. The tackler holds onto the receiver, and the two move off together with a velocity of +2.6 m/s. The mass of the tackler is 115 kg. Assuming that momentum is conserved, find the mass of the receiver. (Cutnell 7.25) **84 kg**
6. A 1055-kg van, stopped at a traffic light, is hit directly in the rear by a 715-kg car traveling with a velocity of +2.25 m/s. Assume that the transmission of the van is in neutral, the brakes are not being applied, and the collision is elastic. What is the final velocity of (a) the car and (b) the van? (Cutnell 7.26) **−0.432 m/s, 1.82 m/s**
7. A cue ball (mass = 0.165 kg) is at rest on a frictionless pool table. The ball is hit dead center by a pool stick, which applies an impulse of +1.50 Ns to the ball. The ball then slides along the table and makes an elastic head-on collision with a second ball of equal mass that is initially at rest. Find the velocity of the second ball just after it is struck. (Cutnell 7.29) **9.09 m/s**
8. A 5.00-kg ball, moving to the right at a velocity of +2.00 m/s on a frictionless table, collides head-on with a stationary 7.50-kg ball. Find the final velocities of the balls if the collision is (a) elastic and (b) completely inelastic. (Cutnell 7.31) **−0.4 m/s, 1.6 m/s; 0.8 m/s**
9. A mine car, whose mass is 440 kg, rolls at a speed of 0.50 m/s on a horizontal track, as the drawing shows. A 150-kg chunk of coal has a speed of 0.80 m/s when it leaves the chute. Determine the velocity of the car/coal system after the coal has come to rest in the car. (Cutnell 7.34) **0.56 m/s**



10. A 30,000-kg freight car is coasting at 0.850 m/s with negligible friction under a hopper that dumps 110,000 kg of scrap metal into it. (a) What is the final velocity of the loaded freight car? (b) How much kinetic energy is lost? (OpenStax 8.36) **0.182 m/s,** $\mathbf{8.52 \times 10^3}$ **J**
11. During an ice show, a 60.0-kg skater leaps into the air and is caught by an initially stationary 75.0-kg skater. (a) What is their final velocity assuming negligible friction and that the 60.0-kg skater's original horizontal velocity is 4.00 m/s? (b) How much kinetic energy is lost? (OpenStax 8.32) **1.78 m/s, −267 J**
12. An automobile has a mass of 2100 kg and a velocity of +17 m/s. It makes a rear-end collision with a stationary car whose mass is 1900 kg. The cars lock bumpers and skid off together with the wheels locked. (a) What is the velocity of the two cars just after the collision? (b) Find the impulse (magnitude and direction) that acts on the skidding cars from just after the collision until they come to a halt. (c) Review: If the coefficient of kinetic friction between the wheels of the cars and the pavement is $\mu_k = 0.68$, determine how far the cars skid before coming to rest. (Cutnell 7.33) **8.9 m/s,** $\mathbf{-3.6 \times 10^4}$ **Ns, 6.0 m**

Physics 04-03 Elastic and Inelastic Collisions	Name: ______________________
Worked-Out Solutions	

1. No, the total KE of all the objects is the same before and after the collision.
2. An elastic collision means the total KE of all the objects is the same before and after the collision.
3. An inelastic collision means the total KE of all the objects is not the same before and after the collision. A perfectly inelastic collision means the total KE of all the objects is not the same before and after the collision.
4. 0, The total momentum before was 0, so the total momentum afterwards is 0.
5. $m_1v_{10} + m_2v_{20} = m_1v_{1f} + m_2v_{2f} \rightarrow m_1\left(0\frac{m}{s}\right) + (115\ kg)\left(4.5\frac{m}{s}\right) = m_1\left(2.6\frac{m}{s}\right) + (115\ kg)\left(2.6\frac{m}{s}\right) \rightarrow$
   $517.5\ kg\frac{m}{s} = m_1\left(2.6\frac{m}{s}\right) + 299\ kg\frac{m}{s} \rightarrow 218.5\ kg\frac{m}{s} = m_1\left(2.6\frac{m}{s}\right) \rightarrow m_1 = \mathbf{84\ kg}$
6. Momentum: $m_1v_{10} + m_2v_{20} = m_1v_{1f} + m_2v_{2f} \rightarrow (1055\ kg)\left(0\frac{m}{s}\right) + (715\ kg)\left(2.25\frac{m}{s}\right) = (1055\ kg)v_{vf} + (715\ kg)v_{cf} \rightarrow$
   $2.25\frac{m}{s} = 1.4755v_{vf} + v_{cf} \rightarrow v_{cf} = 2.25\frac{m}{s} - 1.4755v_{vf}$
   KE: $\frac{1}{2}m_1v_{10}^2 + \frac{1}{2}m_2v_{20}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \rightarrow m_1v_{10}^2 + m_2v_{20}^2 = m_1v_{1f}^2 + m_2v_{2f}^2 \rightarrow$
   $(1055\ kg)\left(0\frac{m}{s}\right)^2 + (715\ kg)\left(2.25\frac{m}{s}\right)^2 = (1055\ kg)v_{vf}^2 + (715\ kg)v_{cf}^2 \rightarrow 5.0625\frac{m^2}{s^2} = 1.4755v_{vf}^2 + v_{cf}^2$
   Substitute: $5.0625\frac{m^2}{s^2} = 1.4755v_{vf}^2 + \left(2.25\frac{m}{s} - 1.4755v_{vf}\right)^2 \rightarrow$
   $5.0625\frac{m^2}{s^2} = 1.4755\ v_{vf}^2 + 5.0625\frac{m^2}{s^2} - \left(6.63975\frac{m}{s}\right)v_{vf} + 2.1771v_{vf}^2 \rightarrow 0 = 3.6526v_{vf}^2 - \left(6.63975\frac{m}{s}\right)v_{vf} \rightarrow$
   $0 = v_{vf}\left(3.6526v_{vf} - 6.63975\frac{m}{s}\right) \rightarrow 0 = 3.6526v_{vf} - 6.63975\frac{m}{s} \rightarrow v_{vf} = \mathbf{1.82\frac{m}{s}}$
   $v_{cf} = 2.25\frac{m}{s} - 1.4755\left(1.82\frac{m}{s}\right) \rightarrow v_{cf} = \mathbf{-0.432\frac{m}{s}}$
7. First ball: $J = mv_f - mv_0 \rightarrow 1.50\ Ns = (0.165\ kg)v_f - (0.165\ kg)\left(0\frac{m}{s}\right) \rightarrow v_f = 9.09\frac{m}{s}$
   Collision: Momentum: $m_1v_{10} + m_2v_{20} = m_1v_{1f} + m_2v_{2f} \rightarrow$
   $(0.165\ kg)\left(9.09\frac{m}{s}\right) + (0.165\ kg)\left(0\frac{m}{s}\right) = (0.165\ kg)v_{1f} + (0.165\ kg)v_{2f} \rightarrow 9.09\frac{m}{s} = v_{1f} + v_{2f} \rightarrow v_{1f} = 9.09\frac{m}{s} - v_{2f}$
   Kinetic Energy: $\frac{1}{2}m_1v_{10}^2 + \frac{1}{2}m_2v_{20}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \rightarrow$
   $\frac{1}{2}(0.165\ kg)\left(9.09\frac{m}{s}\right)^2 + \frac{1}{2}(0.165\ kg)\left(0\frac{m}{s}\right)^2 = \frac{1}{2}(0.165\ kg)v_{1f}^2 + \frac{1}{2}(0.165\ kg)v_{2f}^2 \rightarrow 82.64\frac{m^2}{s^2} = v_{1f}^2 + v_{2f}^2 \rightarrow$
   Substitution: $82.64\frac{m^2}{s^2} = \left(9.09\frac{m}{s} - v_{2f}\right)^2 + v_{2f}^2 \rightarrow 82.64\frac{m^2}{s^2} = 82.64\frac{m^2}{s^2} - \left(18.18\frac{m}{s}\right)v_{2f} + v_{2f}^2 + v_{2f}^2 \rightarrow$
   $0 = 2v_{2f}^2 - \left(18.18\frac{m}{s}\right)v_{2f} \rightarrow 0 = 2v_{2f}^2\left(v_{2f} - 9.09\frac{m}{s}\right) \rightarrow v_{2f} - 9.09\frac{m}{s} = 0 \rightarrow v_{2f} = \mathbf{9.09\frac{m}{s}}$
8. (a) Momentum: $m_1v_{10} + m_2v_{20} = m_1v_{1f} + m_2v_{2f} \rightarrow$
   $(5.00\ kg)\left(2.00\frac{m}{s}\right) + (7.50\ kg)\left(0\frac{m}{s}\right) = (5.00\ kg)v_{1f} + (7.50\ kg)v_{2f} \rightarrow 2.00\frac{m}{s} = v_{1f} + 1.5v_{2f} \rightarrow v_{1f} = 2.00\frac{m}{s} - 1.5v_{2f}$
   Kinetic Energy: $\frac{1}{2}m_1v_{10}^2 + \frac{1}{2}m_2v_{20}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \rightarrow$
   $\frac{1}{2}(5.00\ kg)\left(2.00\frac{m}{s}\right)^2 + \frac{1}{2}(7.50\ kg)\left(0\frac{m}{s}\right)^2 = \frac{1}{2}(5.00\ kg)v_{1f}^2 + \frac{1}{2}(7.50\ kg)v_{2f}^2 \rightarrow 4\frac{m^2}{s^2} = v_{1f}^2 + 1.5v_{2f}^2$
   Substitution: $4\frac{m^2}{s^2} = \left(2\frac{m}{s} - 1.5\ v_{2f}\right)^2 + 1.5v_{2f}^2 \rightarrow 4\frac{m^2}{s^2} = 4\frac{m^2}{s^2} - 6\frac{m}{s}v_{2f} + 2.25v_{2f}^2 + 1.5v_{2f}^2 \rightarrow 0 = 3.75v_{2f}^2 - 6\frac{m}{s}v_{2f} \rightarrow$
   $0 = v_{2f}\left(3.75\ v_{2f} - 6\frac{m}{s}\right) \rightarrow 3.75v_{2f} - 6\frac{m}{s} = 0 \rightarrow v_{2f} = \mathbf{1.6\frac{m}{s}}$
   $v_{1f} = 2.00\frac{m}{s} - 1.5\left(1.6\frac{m}{s}\right) \rightarrow v_{1f} = \mathbf{-0.4\frac{m}{s}}$
   (b) Momentum: $m_1v_{10} + m_2v_{20} = m_1v_{1f} + m_2v_{2f} \rightarrow$
   $(5.00\ kg)\left(2.00\frac{m}{s}\right) + (7.50\ kg)\left(0\frac{m}{s}\right) = (5.00\ kg)v_f + (7.50\ kg)v_f \rightarrow 10\ kg\frac{m}{s} = (12.50\ kg)v_f \rightarrow v_f = \mathbf{0.8\frac{m}{s}}$
9. $m_1v_{10} + m_2v_{20} = m_1v_{1f} + m_2v_{2f} \rightarrow (440\ kg)\left(0.50\frac{m}{s}\right) + (150\ kg)\left(0.80\frac{m}{s}\cos 25°\right) = (440\ kg)v_f + (150\ kg)v_f \rightarrow$
   $328.76\ kg\frac{m}{s} = (590\ kg)v_f \rightarrow v_f = \mathbf{0.56\frac{m}{s}}$

10. (a) $m_1v_{10} + m_2v_{20} = m_1v_{1f} + m_2v_{2f} \rightarrow$

$(30000\ kg)\left(0.850\frac{m}{s}\right) + (110000\ kg)\left(0\frac{m}{s}\right) = (30000\ kg)v_f + (110000\ kg)v_f \rightarrow 25500\ kg\frac{m}{s} = (140000\ kg)v_f \rightarrow$

$v_f = \mathbf{0.182\frac{m}{s}}$

(b) $\Delta KE = \left(\frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2\right) - \left(\frac{1}{2}m_1v_{10} + \frac{1}{2}m_2v_{20}\right) \rightarrow$

$\Delta KE = \left(\frac{1}{2}(30000\ kg)\left(0.182\frac{m}{s}\right)^2 + \frac{1}{2}(110000\ kg)\left(0.182\frac{m}{s}\right)^2\right) - \left(\frac{1}{2}(30000\ kg)\left(0.850\frac{m}{s}\right)^2 + \frac{1}{2}(110000\ kg)\left(0\frac{m}{s}\right)^2\right)$

$\Delta KE = \mathbf{-8.52 \times 10^3\ J}$

11. (a) $m_1v_{10} + m_2v_{20} = m_1v_{1f} + m_2v_{2f} \rightarrow$

$(60\ kg)\left(4\frac{m}{s}\right) + (75\ kg)\left(0\frac{m}{s}\right) = (60\ kg)v_f + (75\ kg)v_f \rightarrow 240\ kg\frac{m}{s} = (135\ kg)v_f \rightarrow v_f = \mathbf{1.78\frac{m}{s}}$

(b) $\Delta KE = \left(\frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2\right) - \left(\frac{1}{2}m_1v_{10} + \frac{1}{2}m_2v_{20}\right) \rightarrow$

$\Delta KE = \left(\frac{1}{2}(60\ kg)\left(1.78\frac{m}{s}\right)^2 + \frac{1}{2}(75\ kg)\left(1.78\frac{m}{s}\right)^2\right) - \left(\frac{1}{2}(60\ kg)\left(4\frac{m}{s}\right)^2 + \frac{1}{2}(75\ kg)\left(0\frac{m}{s}\right)^2\right)$

$\Delta KE = \mathbf{-267\ J}$

12. (a) $m_1v_{10} + m_2v_{20} = m_1v_{1f} + m_2v_{2f} \rightarrow$

$(2100\ kg)\left(17\frac{m}{s}\right) + (1900\ kg)\left(0\frac{m}{s}\right) = (2100\ kg)v_f + (1900\ kg)v_f \rightarrow 35700\ kg\frac{m}{s} = (4000\ kg)v_f \rightarrow v_f = \mathbf{8.9\frac{m}{s}}$

(b) $J = \Delta p \rightarrow J = (m_1v_{1f} + m_2v_{2f}) - (m_1v_{10} + m_2v_{20}) \rightarrow$

$J = \left((2100\ kg)\left(0\frac{m}{s}\right) + (1900\ kg)\left(0\frac{m}{s}\right)\right) - \left((2100\ kg)\left(8.9\frac{m}{s}\right) + (1900\ kg)\left(8.9\frac{m}{s}\right)\right) \rightarrow J = \mathbf{-3.6 \times 10^4\ Ns}$

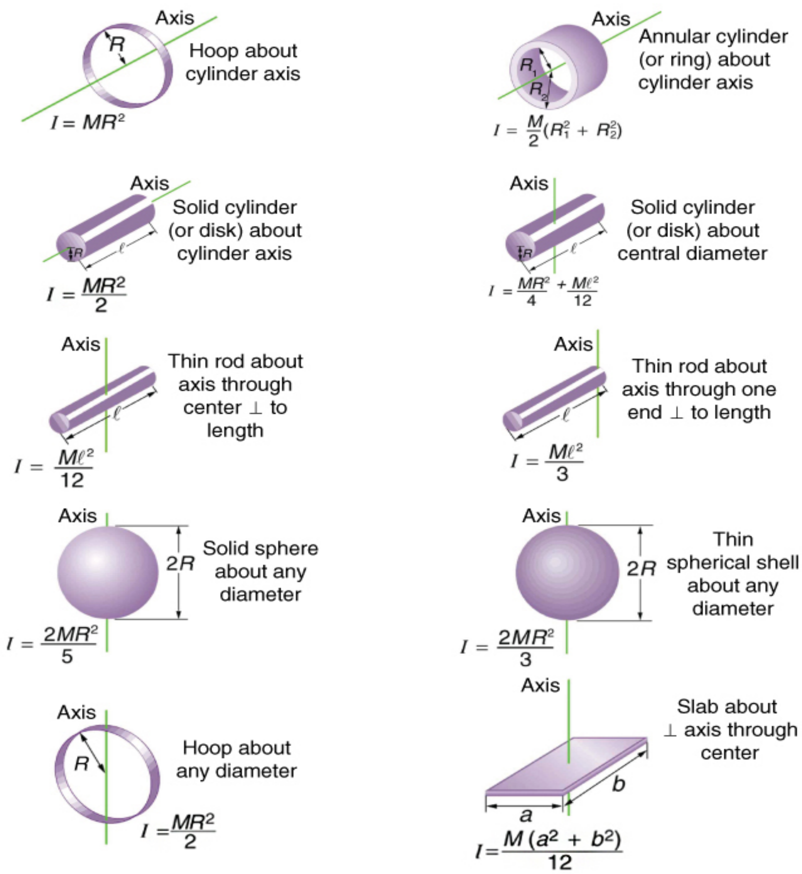
(c) y: $F_{net} = ma \rightarrow N - W = 0 \rightarrow N = W = mg$

x: $F_{net} = ma \rightarrow -\mu N = ma \rightarrow -\mu mg = ma \rightarrow -\mu g = a \rightarrow -(0.68)\left(9.8\frac{m}{s^2}\right) = a \rightarrow a = -6.664\frac{m}{s^2}$

$v_f^2 = v_0^2 + 2a(x - x_0) \rightarrow \left(0\frac{m}{s}\right)^2 = \left(8.9\frac{m}{s}\right)^2 + 2\left(-6.664\frac{m}{s^2}\right)(x) \rightarrow -79.66\frac{m^2}{s^2} = \left(-13.328\frac{m}{s^2}\right)x \rightarrow x = \mathbf{6.0\ m}$

Physics 04-04 Angular Momentum   Name: ____________

Angular Momentum



Linear momentum

- ____________

Angular momentum: $I = MR^2$

- ____________
- Unit: ____________
- ω must be in ____________
- When you rotate something you exert a ____________.
- ____________ torque = faster ____________ in ____________ momentum
- $\tau_{net} = \frac{\Delta L}{\Delta t}$
  - Like $F = \frac{\Delta p}{\Delta t}$

Conservation of Momentum

Linear momentum of a system is ____________ if ____________

- $p_0 = p_f$

Angular momentum of a system is also ____________ if ____________

- $L_0 = L_f$

A 10-kg solid disk with r = 0.40 m is spinning at 8 rad/s. A 9-kg solid disk with r = 0.30 m is dropped onto the first disk. If the second disk was initially not rotating, what is the angular velocity after the disks are together?

What was the torque applied by the first disk onto the second if the collision takes 0.01 s?

- Angular Momentum conserved if net external ____________ is ____________
- Linear Momentum conserved if net external ____________ is ____________
- Kinetic Energy conserved if ____________ collision

Direction of angular quantities

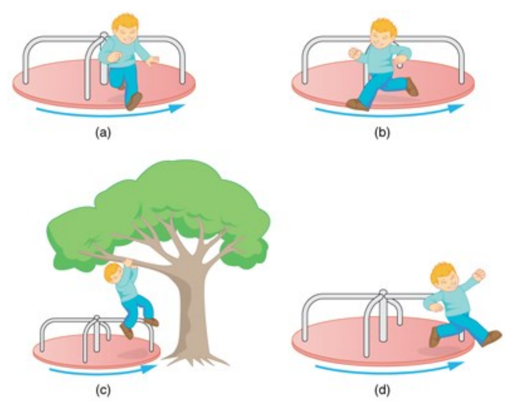
Right-hand Rule

- Hold hand out with ____________ out along ____________
- Curl your ____________ in direction of ____________ (you may have to turn your hand upside down)
- Vector in ____________ of ____________

Physics 04-04 Angular Momentum	*Name: ________________________*

A person is holding a spinning bicycle wheel while he stands on a stationary frictionless turntable. What will happen if he suddenly flips the bicycle wheel over so that it is spinning in the opposite direction?

Practice Work



1. Suppose a child walks from the outer edge of a rotating merry-go-round to the inside. Does the angular velocity of the merry-go-round increase, decrease, or remain the same? Explain your answer.
2. Helicopters have a small propeller on their tail to keep them from rotating in the opposite direction of their main lifting blades. Explain in terms of Newton's third law why the helicopter body rotates in the opposite direction to the blades.
3. When there is a global heating trend on Earth, the atmosphere expands and the length of the day increases very slightly. Explain why the length of a day increases.
4. Nearly all conventional piston engines have flywheels on them to smooth out engine vibrations caused by the thrust of individual piston firings. Why does the flywheel have this effect?
5. Suppose an ice hockey puck strikes a hockey stick that lies flat on the ice and is free to move in any direction. Which quantities are likely to be conserved: angular momentum, linear momentum, or kinetic energy (assuming the puck and stick are very resilient)?
6. (a) Calculate the angular momentum of the Earth in its orbit around the Sun. (b) Compare this angular momentum with the angular momentum of Earth on its axis. (OpenStax 10.36) $\mathbf{2.66 \times 10^{40}\ kg \cdot \frac{m^2}{s}, 7.07 \times 10^{33}\ kg \cdot \frac{m^2}{s}}$
7. (a) What is the angular momentum of the Moon in its orbit around Earth? (b) How does this angular momentum compare with the angular momentum of the Moon on its axis? Remember that the Moon keeps one side toward Earth at all times.(OpenStax 10.37) $\mathbf{2.89 \times 10^{34}\ kg \cdot \frac{m^2}{s}, 2.37 \times 10^{29}\ kg \cdot \frac{m^2}{s}}$
8. Suppose you start an antique car by exerting a force of 300 N on its crank for 0.250 s. What angular momentum is given to the engine if the handle of the crank is 0.300 m from the pivot and the force is exerted to create maximum torque the entire time? (OpenStax 10.38) $\mathbf{22.5\ kg \cdot m^2/s}$
9. A playground merry-go-round has a mass of 120 kg and a radius of 1.80 m and it is rotating with an angular velocity of 0.500 rev/s. What is its angular velocity after a 22.0-kg child gets onto it by grabbing its outer edge? The child is initially at rest. (OpenStax 10.39) $\mathbf{2.30\ rad/s}$
10. Three children are riding on the edge of a merry-go-round that is 100 kg, has a 1.60-m radius, and is spinning at 20.0 rpm. The children have masses of 22.0, 28.0, and 33.0 kg. If the child who has a mass of 28.0 kg moves to the center of the merry-go-round, what is the new angular velocity in rpm? (OpenStax 10.40) **25.3 rpm**
11. (a) Calculate the angular momentum of an ice skater spinning at 6.00 rev/s given his moment of inertia is 0.400 kg · m^2. (b) He reduces his rate of spin (his angular velocity) by extending his arms and increasing his moment of inertia. Find the value of his moment of inertia if his angular velocity decreases to 1.25 rev/s. (c) Suppose instead he keeps his arms in and allows friction of the ice to slow him to 3.00 rev/s. What average torque was exerted if this takes 15.0 s? (OpenStax 10.41) $\mathbf{15.1\ kg \cdot \frac{m^2}{s},\ 1.92\ kg \cdot m^2,\ -0.503\ N \cdot m}$

Physics Unit 4: Momentum Review

1. Know about impulse, momentum, Law of Conservation of Momentum, elastic, inelastic, kinetic energy, right-hand rule, angular momentum, moment of inertia
2. Know how to minimize force during a collision
3. What does changing the radius where the mass is located, change the angular momentum of an object?
4. Why is the total momentum of a system conserved when there is no net force on the system?
5. A 500-kg car runs into a tree. If it was traveling at 8 m/s right before it hit and the collision lasted 0.01 s, what was the average force between the car and the tree?
6. A 0.5-kg ball traveling at 20 m/s is hit by a baseball bat. After it is hit, it is traveling at 5 m/s in the opposite direction. What impulse was delivered to the ball?
7. A penguin is waddling at 0.5 m/s. If the penguin weighs 50 N, what is the magnitude of its momentum?
8. A 10-kg pitching machine at rest contains a 0.5-kg ball. When pitched, the ball leaves the machine with a speed of 40 m/s. What is the recoil speed of the machine?
9. A 1500-kg car traveling east at 10 m/s collides with a 1000-kg car traveling west at 12 m/s. The cars stick together after the collision. What is their common speed after the collision?
10. A 0.2-kg bird is flying 50 m above the ground at 15 m/s. What is the bird's kinetic energy?
11. Two identical marbles bounce off each other. The first marble's mass is 0.02 kg, its initial velocity was 3 m/s, and its final velocity was –2 m/s. The second marble's mass is 0.02 kg, its initial velocity was –1 m/s and its final velocity was 4 m/s. What type of collision occurred?
12. A 1200-kg car is stopped at a stop sign when it is hit in the back by a 2500-kg van. The two vehicles stick together after the accident. From the skid marks and measured coefficient of friction you calculate that the speed of both cars combined after the collision is 11 m/s. What was the speed of the van before the collision?
13. A ring is spinning on its central axis. Its mass is 2 kg, internal radius 0.8 m, and external radius is 1 m. What is its angular momentum when it is spinning at π rad/s?
14. A 5-kg solid disk is rotating around its axis at 13 rad/s when a second nonrotating solid disk with mass 2 kg is dropped on top so that they share the same axis. The first disk has radius 1 m and the second disk has radius 0.7 m. What is the final angular speed of disks?

Physics Unit 4: Momentum Review

Answers

2. $F\Delta t = mv_2 - mv_1$: Increase the time of the collision or decrease the final speed by not letting it rebound
3. The angular momentum is related to the moment of inertia by $L = I\omega$. The moment of inertia is directly proportional to R^2. Increasing the radius increases the moment of inertia which increases the angular momentum.
4. $F\Delta t = mv_2 - mv_1$ If $F = 0$, then the change in momentum = 0.
5. $F\Delta t = mv_2 - mv_1$

   $F(0.01\ s) = (500\ kg)\left(0\frac{m}{s}\right) - (500\ kg)\left(8\frac{m}{s}\right)$

   $F = \mathbf{-400{,}000\ N}$
6. $F\Delta t = mv_2 - mv_1$

   $J = F\Delta t$

   $J = mv_2 - mv_1$

   $J = (0.5\ kg)\left(-5\frac{m}{s}\right) - (0.5\ kg)\left(20\frac{m}{s}\right)$

   $J = \mathbf{12.5\ kg\ m/s}$
7. $p = mv$

   $w = mg$

   $50\ N = m\left(9.8\frac{m}{s^2}\right)$

   $5.1\ kg = m$

   $p = (5.1\ kg)\left(0.5\frac{m}{s}\right) = \mathbf{2.55\frac{m}{s}}$
8. $m_b v_{b1} + m_m v_{m1} = m_b v_{b2} + m_m v_{m2}$

   $(0.5\ kg)\left(0\frac{m}{s}\right) + (10\ kg)\left(0\frac{m}{s}\right) = (0.5\ kg)\left(40\frac{m}{s}\right) + (10kg)v_{m2}$

   $0 = 20\ kg\frac{m}{s} + (10\ kg)v_{m2}$

   $-20\ kg\frac{m}{s} = (10\ kg)v_{m2}$

   $\mathbf{-2\frac{m}{s}} = v_{m2}$
9. $m_1 v_{11} + m_2 v_{21} = m_1 v_{12} + m_2 v_{22}$

   $(1500\ kg)\left(10\frac{m}{s}\right) + (1000\ kg)\left(-12\frac{m}{s}\right) = (1500\ kg)v_f + (1000\ kg)v_f$

   $3000\ kg\frac{m}{s} = (2500\ kg)v_f$

   $\mathbf{1.2\frac{m}{s}} = v_f$
10. $KE = \frac{1}{2}mv^2$

    $KE = \frac{1}{2}(0.2\ kg)\left(15\frac{m}{s}\right)^2 = \mathbf{22.5\ J}$
11. Check for conservation of KE

    $\frac{1}{2}m_1 v_{11}^2 + \frac{1}{2}m_2 v_{21}^2 = \frac{1}{2}m_1 v_{12}^2 + \frac{1}{2}m_2 v_{22}^2$

    $\frac{1}{2}(0.02\ kg)\left(3\frac{m}{s}\right)^2 + \frac{1}{2}(0.02\ kg)\left(-2\frac{m}{s}\right)^2 =$

    $\frac{1}{2}(0.02\ kg)\left(-1\frac{m}{s}\right)^2 + \frac{1}{2}(0.02\ kg)\left(4\frac{m}{s}\right)^2$

    $0.13\ J \neq 0.17\ J$

    KE is not conserved, so the collision is **inelastic**.
12. $m_c v_{c1} + m_v v_{v1} = m_c v_{c2} + m_v v_{v2}$

    $(1200\ kg)\left(0\frac{m}{s}\right) + (2500\ kg)v_{v1} =$

    $(1200\ kg)\left(11\frac{m}{s}\right) + (2500\ kg)\left(11\frac{m}{s}\right)$

    $(2500\ kg)v_{v1} = 40700\ kg\ m/s$

    $v_{v1} = \mathbf{16.3\frac{m}{s}}$
13. $L = I\omega$

    $I = \frac{M}{2}(R_1^2 + R_2^2)$

    $I = \frac{2\ kg}{2}((0.8\ m)^2 + (1\ m)^2) = 1.64\ kg\ m^2$

    $L = (1.64\ kg\ m^2)\left(\pi\frac{rad}{s}\right) = \mathbf{5.15\ kg\frac{m^2}{s}}$
14. $I_1\omega_{11} + I_2\omega_{21} = I_1\omega_{12} + I_2\omega_{22}$

    $I_1 = \frac{MR^2}{2} = \frac{(5\ kg)(1\ m)^2}{2} = 2.5\ kg\ m^2$

    $I_2 = \frac{MR^2}{2} = \frac{(2\ kg)(0.7\ m)^2}{2} = 0.49\ kg\ m^2$

    $(2.5\ kg\ m^2)\left(13\frac{rad}{s}\right) + (0.49\ kg\ m^2)\left(0\frac{m}{s}\right) =$

    $(2.5\ kg\ m^2)\omega_f + (0.49\ kg\ m^2)\omega_f$

    $32.5\ kg\frac{m^2}{s} = (2.99\ kg\ m^2)\omega_f$

    $\mathbf{10.9\frac{rad}{s}} = \omega_f$